

**Vermont Grade Expectations
Grade Two
Support Resources
April 2008**

This document contains items keyed to the Vermont Grade Expectations for grade two and are designed to help clarify the Grade Expectations. While the items exemplify aspects of each GE, they may not illustrate all skills and/or concepts included in the GE. ***Furthermore, the items are not designed to be used as an assessment of the GE, but serve the purpose of exemplifying the concepts and skills within the GE. Because this is not an assessment document, the items should not be copied and used for assessment purposes.*** The examples are provided to support teachers in understanding the essence of the GE, and are a suggested resource that could be used for the teaching and learning of a concept.

Appendix A contains web links when items were drawn from outside sources. Many of these sites contain additional materials which may be useful in the instructional setting. Appendix B contains the Depth of Knowledge descriptors based on Norman Webb's work, and Appendix C contains the answers and Depth of Knowledge classification for each item. In addition, each item is numbered sequentially in the upper left hand corner for ease of reference to the answers in Appendix C.

Number and Operations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

GRADE 2 SUPPORT MATERIALS

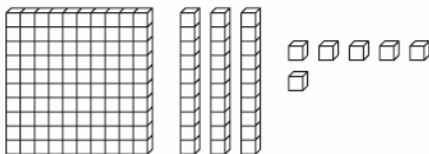
M2: 1 Demonstrates conceptual understanding of rational numbers with respect to: whole numbers from 0 to 199 using place value, by applying the concepts of equivalency in composing or decomposing numbers (e.g., $34 = 17 + 17$; $34 = 29 + 5$); and in expanded notation (e.g., $141 = 1 \text{ hundred} + 4 \text{ tens} + 1 \text{ one}$ or $141 = 100 + 40 + 1$) **using models, explanations, or other representations**; and

positive fractional numbers (benchmark fractions: $a/2$, $a/3$, or $a/4$, where a is a whole number greater than 0 and less than or equal to the denominator) as a part to whole relationship in area and set models where the denominator is equal to the number of parts in the whole **using models, explanations, or other representations**.

M(N&O)-2-1

1

1 Look at these blocks.



Demonstrates conceptual understanding of rational numbers with respect to whole numbers by applying the concepts of equivalency in composing and decomposing numbers using models

What is the value of these blocks?

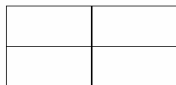
- A. 1 ten + 3 hundreds + 6 ones
- B. 1 hundred + 3 tens + 6 ones
- C. 100 hundreds + 30 tens + 6 ones
- D. 10 tens + 3 hundreds + 6 ones

NECAP Released Item 2005

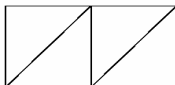
2

K8. Which rectangle is NOT divided into 4 equal parts?

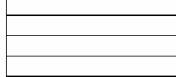
A.



B.



C.



D.



Demonstrates conceptual understanding of positive fractional numbers as a part to whole relationship in area models

Show or explain how you know.

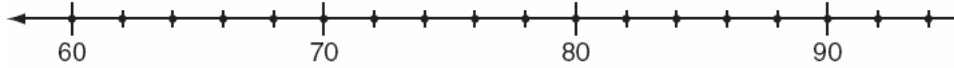
Adapted from New Hampshire Department of Education

M2: 2 Demonstrates understanding of the relative magnitude of numbers from 0 to 199 by ordering whole numbers; by comparing whole numbers to each other or to benchmark whole numbers (10, 25, 50, 75, 100, 125, 150, or 175); by demonstrating an understanding of the relation of inequality when comparing whole numbers by using “1 more,” “1 less,” “10 more,” “10 less,” “100 more,” or “100 less”; or by connecting number words and numerals to the quantities they represent using models, number lines, or explanations.

M(N&O)–2–2

3

11 Look at this number line.



Demonstrates understanding of the relative magnitude of numbers by demonstrating an understanding of the relation of inequality when comparing numbers using “10 more”

Put an “X” on the point that represents 10 more than 78.

NECAP Released Item 2006

M2: 3 Demonstrates conceptual understanding of mathematical operations involving addition and subtraction of whole numbers by solving problems involving joining actions, separating actions, part-part-whole relationships, and comparison situations; and addition of multiple one-digit whole numbers. (See Appendix A.)

4

**Classification of Word Problems
Appendix A Vermont Grade Expectations**

Addition and Subtraction

Problem Type			
Join	<i>(Result Unknown)</i> Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether?	<i>(Change Unknown)</i> Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether?	<i>(Start Unknown)</i> Connie had some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?
Separate	<i>(Result Unknown)</i> Connie had 13 marbles. She gave 5 to Juan. How many marbles does Connie have left?	<i>(Change Unknown)</i> Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give to Juan?	<i>(Start Unknown)</i> Connie had some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with?
Part-Part-Whole	<i>(Whole Unknown)</i> Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?	<i>(Part Unknown)</i> Connie has 13 marbles. 5 are red and the rest are blue. How many blue marbles does Connie have?	
Compare	<i>(Difference Unknown)</i> Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan?	<i>(Compare Quantity Unknown)</i> Juan has 5 marbles. Connie has 8 more than Juan. How many marbles does Connie have?	<i>(Referent Unknown)</i> Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have?

Appendix A: Vermont Grade Expectations.

There are 7 boys, 9 girls, and 5 adults at a party. How many people are at the party altogether?

- A. 11
- B. 16
- C. 21
- D. 22

Demonstrates conceptual understanding of mathematical operations involving addition of multiple one-digit whole numbers

NECAP Released item 2006

M2: 4 No **M2: 4** at this grade level

M2: 5 Demonstrates understanding of monetary value by adding coins together to a value no greater than \$1.99 and representing the result in dollar notation; making change from \$1.00 or less, or recognizing equivalent coin representations of the same value (values up to \$1.99).

M(N&O)-2-5

The table below shows how many coins each child in the Jones family has.

Children's Coins

Child	Coins
Byron	3 quarters
Pam	100 pennies
Linda	4 quarters
William	50 pennies

Demonstrates understanding of monetary value by adding coins together to a value no greater than \$1.99 and representing the result in dollar notation; recognizing equivalent coin representations of the same value (values up to \$1.99)

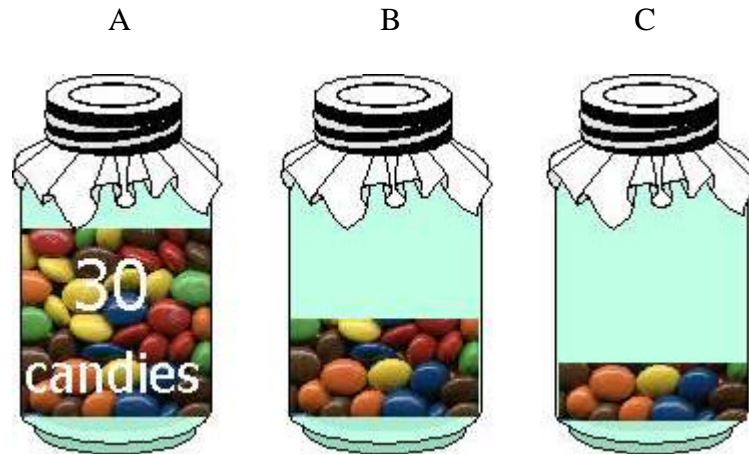
Using dollar notation, show how much each child has. Which two children have the same amount of money?

Adapted from Massachusetts Comprehensive Assessment Released Items Grade 2

M2: 6 Mentally adds and subtracts whole-numbers facts through twenty with accuracy.

M2: 7 Estimates and evaluates the reasonableness of solutions appropriate to grade level.

7



Estimates and evaluates the reasonableness of solutions.

John said he believes jar B has 25 candies and jar C has 20 candies. Use pictures or words to show if John is correct.

Adapted from New Hampshire Department of Education Item Fitting Committee 2006

Geometry and Measurement

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills. We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes. Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world. In each area, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

M2: 9 Uses properties, attributes, composition, or decomposition to sort or classify polygons or objects by a combination of two or more nonmeasurable or measurable attributes.

M(G&M)-2-1

8

Color, Shape, and Size

All the blocks inside have the same shape, color, or size. Some of the blocks outside belong with the blocks inside. Which blocks belong inside?

Uses properties, attributes, to sort or classify polygons or objects by a combination of two or more nonmeasurable or measurable attributes

National Library of Virtual Manipulatives

9

Look at the trapezoid below. 



Uses composition to classify polygons.

Use **two different** shapes from your envelope to make this trapezoid. Trace the shapes to show how they fit together to make this trapezoid.

NECAP Released Item 2006

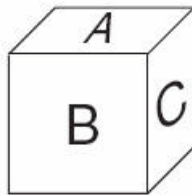
M2: 10 No M2: 10 at this grade level

M2: 11 Identifies three-dimensional shapes (rectangular prisms, triangular prisms, cylinders, or spheres) and their attributes and recognizes them in their environment.

10 Which object is a cylinder?



A



C



B



D

Identifies 3 dimensional shapes and recognizes them in their environment.

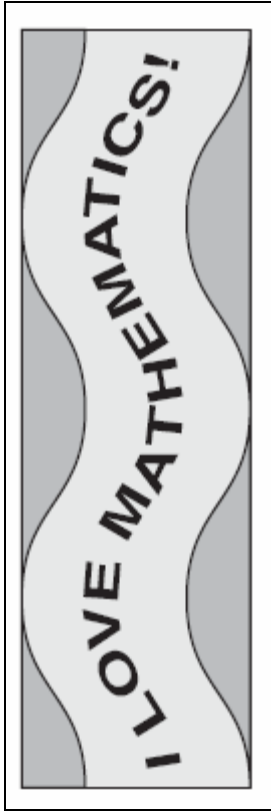
Assessment for the California Mathematics Standards Grade 2

M2: 12 No M2: 12 at this grade level

M2: 13 No M2: 13 at this grade level

M2: 14 Demonstrates conceptual understanding of perimeter and area by using models or manipulatives to surround and cover polygons.

- 11 Use 1 centimeter cubes to cover this bookmark. What is the area of this bookmark?
Use 1 centimeter cubes to surround this bookmark. What is the perimeter of this bookmark?



Demonstrates conceptual understanding of perimeter by using models to surround and cover polygons

Vermont Department of Education – Graphic from NECAP Released Item 2006 Grade 3

M2: 15 Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands. (Benchmarks in Appendix B.)

M(G&M)-2-7

- 12 A movie is 115 minutes long. Which statement about this movie is true?



Makes conversions within systems when solving problems

- A. The movie is less than 2 hours long.
- B. The movie is exactly 2 hours long.
- C. The movie is more than 2 hours long.

NECAP Released Item 2006

M2: 16 Determines elapsed and accrued time as it relates to the patterns of days of the week, months, hours, and tells time to five minutes.

13

[A MOVIE STARTED AT ELEVEN O'CLOCK A.M. AND LASTED THREE HOURS. AT WHAT TIME DID THE MOVIE END?]

12:00 p.m.

1:00 p.m.

2:00 p.m.

3:00 p.m.

Assessment for the California Mathematics Standards Grade 2

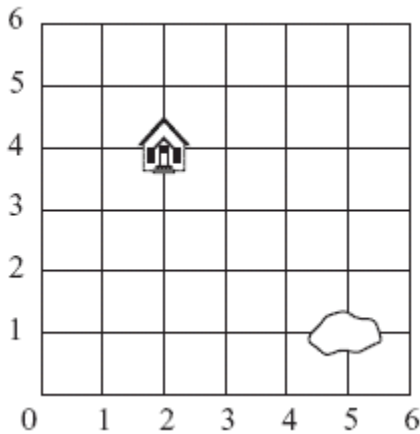
M2: 17 No **M2: 17** at this grade level

M2: 18 Solves problems using a two-dimensional coordinate system (x and y axes—quadrant I) to locate and describe positions on a map.

14

Solves problems using a two dimensional coordinate system (quadrant I) to locate and describe positions on a map

The ordered pair (5,1) shows where the duck pond is on the grid below.



Key	
	Kyle's House
	Duck Pond

Write the ordered pair that shows where Kyle's house is.

Massachusetts Comprehensive Assessment System

Functions and Algebra

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students will progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

2: 19: Identifies and extends to specific cases a variety of patterns (linear and non numeric) represented in models, tables, or sequences by extending the pattern to the next element, or finding a missing element (e.g., 2, 4, 6, . . . 10).

M(F&A)-2-1

15

Look at the pattern shown below.

2, 5, 8, 11, _____,

Identifies and extends a linear pattern represented in a sequence to the next element





The pattern continues in the same way. What is the next number in this pattern?

Adapted from PAWS Sample Item 2006

16 Look at this pattern.



What shape is missing?

- A. 
- B. 
- C. 
- D. 

Identifies and extends a non-numeric pattern represented in sequences to find a missing element

NECAP Practice Test

M2: 20 Demonstrates a conceptual understanding of linear relationships ($y = kx$) **as a constant rate of change** qualitatively (growth—student growing taller) and quantitatively (measurable growth—2 inches each year) change.

17

[DAVID READS TWO PAGES EVERY FIVE MINUTES. HOW MANY PAGES WILL DAVID HAVE READ AFTER TWENTY-FIVE MINUTES?]

David's Reading

Minutes	5	10	15	20	25
Pages	2	4	6	8	

Demonstrates conceptual understanding of linear relationships as a constant rate of change

Adapted from Assessment for the California Mathematics Standards Grade 2

M2: 21 No M2: 21 at this grade level

M2: 22 Demonstrates conceptual understanding of equality by finding the value that will make an open sentence true (e.g., $2 + \square = 7$). (limited to one operation and limited to use addition or subtraction).

M(F&A)–2–4

18

a. Write a number in the box to make this number sentence true.

$$3 + 6 = \square + 5$$

Demonstrates conceptual understanding of equality by finding the value that makes an open sentence true

b. Write the numbers 8, 6, 2, and 4 in the boxes to make this number sentence true. Use each number only once.

$$\square + \square = \square + \square$$

NECAP Released Item 2006

Data, Statistics and Probability

Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

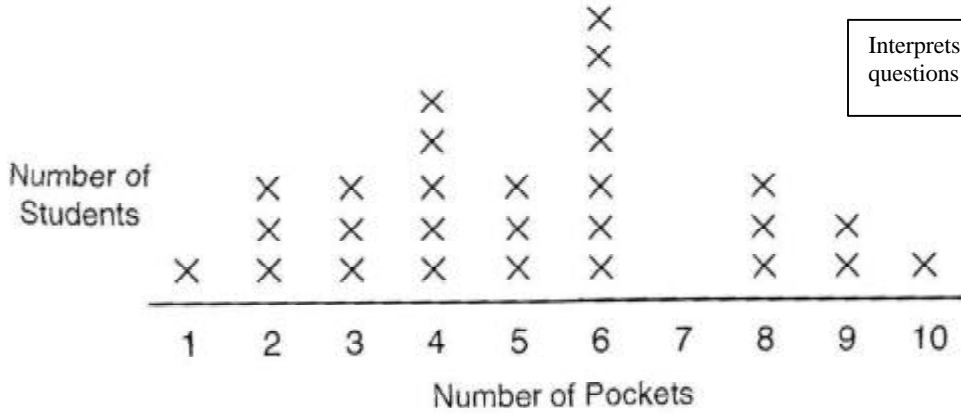
Standard 7.9: Data, Statistics, and Probability Concepts

M2: 23 Interprets a given representation (pictographs with one-to-one correspondence, line plots, tally charts, or tables) to answer questions related to the data, or to analyze the data to formulate conclusions.

(IMPORTANT: Analyzes data consistent with concepts and skills in M2: 24.)

M(DSP)–2–1

19



Interprets a line plot to answer questions related to the data

KEY: X represents 1 student

How many pockets did MOST students have?

National Council of the Teachers of Mathematics, Inc., *Principles and Standards for School Mathematics*, National Council of the Teacher of Mathematics, 2000.

M2: 24 Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using “more,” “less,” or “equal.”

M(DSP)–2–2

20

Look at this tally chart.

Favorite Color

Red	
Blue	
Purple	
Yellow	
Green	

Analyzes distributions in data by determining or using “more”

What colors were the favorites of four or more people?

NECAP Released Item

M2: 25 Organizes and displays data using diagrams, models, tally charts, or tables to answer questions related to the data, to analyze the data to formulate conclusions.

(IMPORTANT: *Analyzes data consistent with concepts and skills in M2: 24.*)

21

Kevin, Jade and Hannah recorded the number of apples they each ate in 1 week. Kevin ate 6 apples, Jade ate 3 apples, and Hannah ate 4 apples.

Make a tally chart or table to show the number of apples Kevin, Jade and Hannah each ate.

How many more apples did Kevin eat than Hannah?

Organizes and displays data using tally charts or tables to answer questions related to the data

Adapted from New York State Testing Program







M2: 26 Uses counting techniques to solve problems involving combinations using a variety of strategies (e.g., student diagrams, organized lists, tables, tree diagrams, or^{sc} others); (e.g., “How many ways can you make 50 cents using nickels, dimes, and quarters?”)

M(DSP)–2–4

22

Mandy is going to wrap a gift. The kinds of wrapping paper and bows she can choose are shown below.

Gift Wrap

Kinds of Wrapping Paper	Kinds of Bows
	
	
	
	

Uses counting techniques to solve problems involving combinations

How many different ways can Mandy choose 1 kind of wrapping paper and 1 kind of bow?

MCAS Released Item

M2: 27 For a probability event in which the sample space may or may not contain equally likely outcomes, uses experimental probability to describe the likelihood or chance of an event using “more likely,” “less likely,” “equally likely,” “certain,” or “impossible.”

23 Toss a coin 10 times. Before tossing the coin, make a guess about how many times you think heads will be on top. Is it more likely, less likely or equally certain that you will toss more heads than tails? Why?

Uses experimental probability to describe the likelihood of an event

Now, toss the coin 10 times and record the outcome from each toss. What was the experimental probability of tossing heads?

Vermont Department of Education

M2: 28 In response to a teacher - or student-generated question or hypothesis, collects appropriate data, organizes the data, displays/represents the data, and makes observations about the data to draw conclusions about the question or hypothesis being tested.

24

What kind of ice cream flavor is the favorite flavor of the children in your class? Make a guess and then collect, organize and display data to find out if your guess was correct. Share one thing you learned about the favorite ice cream flavors of the children in your class.

(IMPORTANT: Analyzes data consistent with concepts and skills in M2: 24.)

Vermont Department of Education

M2: 29 No **M2: 29** at this grade level

Standard 2.5: Mathematical Dimensions,

Standard 7.10: Mathematical Problem Solving and Reasoning—Applications

M2: 30 Demonstrate understanding of mathematical problem solving² and communication through:³

- **Approach & Reasoning**—The reasoning, strategies, and skills used to solve the problem;
- **Connections**—Demonstration of observations, applications, extensions, and generalizations;
- **Solution**—All of the work that was done to solve the problem, including the answer;
- **Mathematical Language**—The use of mathematical language in communicating the solution;
- **Mathematical Representation**—The use of mathematical representation to communicate the solution; and
- **Documentation**—Presentation of the solution.

² Problem-solving situations are mathematical problems that reflect the levels of mathematics in the Grade Level Expectations.

³ See *Vermont*

APPENDIX A

California Department of Education. <http://www.cde.ca.gov/ta/tg/sr/resources.asp>

Delaware Student Testing Program. http://www.doe.state.de.us/AAB/DSTP_items.html

Massachusetts Department of Education. <http://www.doe.mass.edu/mcas/testitems.html>

National Library of Virtual Manipulatives. <http://nlvm.usu.edu/en/nav/vlibrary.html>

NCTM. <http://www.nctm.org/resources/elementary.aspx>

New England Common Assessment.

<http://reporting.measuredprogress.org/NECAPpublicNH/relitem.aspx>

New Hampshire Item Fitting Committee

New York State Assessment. <http://www.nysedregents.org/testing/elementtests.html>

Pennsylvania Department of Education.

http://www.pde.state.pa.us/a_and_t/cwp/view.asp?a=108&q=103267

TIMSS. <http://www.nctm.org/resources/elementary.aspx>

APPENDIX B: Implied Cognitive Demand and Depth of Knowledge

A fundamental criterion used to develop the NECAP GLEs and GSEs is that the expectations should explicitly indicate cognitive demand (how content interacts with process) and that there should be a mix of cognitive demand levels at all grades. One should not assume that students at lower grades do less cognitively demanding work. The cognitive demand or depth of knowledge required by an expectation or an assessment item is related to the number and strength of connections of concepts and procedures that a student needs to make to produce a response, including the level of reasoning required along with self-monitoring. Furthermore, there are additional factors that influence cognitive demand including contextual requirements, language, the number and variety of representations, requirements for generalizations to new situations, and the opportunity to learn.

It is important to note that depth of knowledge is not synonymous with difficulty. As an example, solving a multi-step linear equation with variables on both sides may be a difficult task for middle school students; however, the task can be solved by applying a standard procedure making the task of low complexity.

The NECAP states believe that expectations and assessments should be aligned in terms of their cognitive complexity. That is, the cognitive complexities of the assessment items should match those of the standards (what students are expected to know and be able to do). To ensure this alignment, the NECAP states have adopted Norman L. Webb's (senior researcher with the Wisconsin Center for Educational Research) Depth of Knowledge classification system. Norman Webb's system is based on four levels of classification. The full descriptions of each level are given on pages 4 and 5. The levels can be summarized as follows.

Level 1	Recall
Level 2	Skill/Concept
Level 3	Strategic Thinking
Level 4	Extended Thinking

The NECAP states, together with a committee of educators, analyzed the GLEs and GSEs for their implied cognitive demand. All aspects of each expectation were analyzed and the implied cognitive demand levels were recorded. One of the charges of the NECAP test item review committees is to ensure that assessment items align not only with the expectations but also with their implied cognitive demands. The range of cognitive demands for each GLE and GSE is summarized in Table 1. It should be noted that the highest level listed for each GLE and GSE should be thought of as a "ceiling" not a "target". A NECAP goal is to write items that cover the range of the levels indicated and not just the highest level. If one assesses only at the "target"

level, all GLEs with a level 3 (for example) as their “ceiling” would only be assessed at level 3. This would potentially have two negative impacts on the assessment: 1) The assessment as a whole would be too difficult, and 2) important information about student learning along the achievement continuum would be lost. To the extent possible, each GLE and GSE should be assessed at the “ceiling” and at least one level below the “ceiling” in order to provide additional diagnostic information to educators. Furthermore, Table 2 shows an example of an expectation and how the different aspects of the expectation interact with Table 1.

Vermont Grade Expectations Implied Cognitive Demand and Depth of Knowledge

	K	1st	2nd	3rd	4th	5th	6th	7th	8th	HS
Arithmetic, Numbers and Operations										
M1	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2
M2	1,2	1,2	1	2	2	2	2	2	2	
M3	2	2	1,2	2	2	2,3	2,3	2,3		
M4	1	1,2		1,2,3	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3
M5	1	1	1,2							
M6		1	1	1	1	1	1			
M7	2	2	2	2	2	2	2	2	2	2
M8		1,2	2	2,3	2,3	2,3	2,3	2,3	2,3	2,3
Geometry and Measurement Concepts										
M9	1	1	1,2,3	1, 2	1,2	1,2	1,2	1,2	1,2,3	1,2,3
M10								1,2	1,2,3	
M11		1	1,2	2	1,2	1,2	1,2	2,3		2,3
M12				2	2	1,2	1,2	1,2		
M13					2	2	1,2	1,2,3	1,2,3	1,2,3
M14			1,2	1,2	1,2	1,2	1,2,3	1,2,3	1,2,3	1,2,3
M15	1	1	1	1	1,2	1,2	1,2	1,2	1,2	1,2
M16	1	1	1	1	1	1				
M17								1,2	1,2	1,2
M18	1	1	2	2	2	2	2			
Functions and Algebra Concepts										
M19	1,2	1,2	2	2	2	2	2,3	2,3	2,3	2,3
M20	2	1,2	1,2	2	2	2	1,2	1,2,3	1,2,3	1,2,3
M21					1	1	1,2	1,2	1,2	1,2
M22	1,2	1,2	1	1,2	1,2	1,2	1,2	1,2	1,2,3	1,2,3

Data, Statistics, and Probability Concepts										
M23	2,3	2,3	1,2,3	1,2,3	1,2, 3	1,2,3	1,2,3	1,2,3	2,3	2,3
M24	2,3	2,3	2,3	2,3	2,3	2,3	2,3	2,3	2,3	1,2,3
M25	2,3	2,3	2,3	1,2	2,3	1,2	2,3	2,3	2,3	2,3
M26			2	2	2,3	2,3	2,3	2,3	2,3	1,2,3
M27		1,2	1,2	1,2	1,2	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3
M28	3,4	3,4	3,4	3,4	3,4	3,4	3,4	3,4	3,4	3,4
M29				2,3	1,2,3	1,2,3	2,3	2,3	2,3	2,3
M30	3,4	3,4	3,4	3,4	3,4	3,4	3,4	3,4	3,4	3,4

Note: GEs assessed on the NECAP are shaded gray.

Depth of Knowledge Descriptors for Mathematics
Norman L. Webb
March 28, 2002

Mathematics Depth of Knowledge Levels

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include “identify,” “recall,” “recognize,” “use,” and “measure.” Verbs such as “describe” and “explain” could be classified at different levels depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include “classify,” “organize,” “estimate,” “make observations,” “collect and display data,” and “compare data.” These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as “explain,” “describe,” or “interpret” could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Mathematics Depth of Knowledge Levels continued

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

References

New Hampshire, Rhode Island, and Vermont Department of Education. (2004). *Draft Tri-State New England (TSNE) Mathematics Test Specifications*. New Hampshire, Rhode Island, and Vermont Department of Education.

Webb, L. Norman. (2002). *Depth of Knowledge Levels for Four Content Areas*.

Webb, L. Norman. (1997). *Criteria for Alignment of Expectations and Assessments in Mathematics and Science Education*. Research Monograph No. 8. Council of Chief State School Officers

APPENDIX C:
Answers and DoK Codes for Examples

Item Number	Answer	Depth of Knowledge
1	B	2
2	D	2
3	88	2
4	No Answers	
5	C	1
6	\$.75, \$1.00, \$1.00, \$.50 Pam and William	2
7	Answers will vary	2
8	Large square and rectangle	2
9	Triangle and rhombus	2
10	Soup can	1
11	30 cubes 26 cubes	2
12	A	1
13	2:00 PM	1
14	School	1
15	14	2
16	C	2
17	10	2
18	a) 4 b) $8+2=6+4$	2
19	6	2
20	Red, blue, green	2
21	Makes accurate representation; 2	2
22	8	2
23	Answers will vary	2
24	Answers will vary	3