Vermont Grade Expectations
Grade Four Support Resources
April 2008

This document contains items keyed to the Vermont Grade Expectations for grade four and are designed to help clarify the Grade Expectations. While the items exemplify aspects of each GE, they may not illustrate all skills and/or concepts included in the GE. ***Furthermore, the items are not designed to be used as an assessment of the GE, but serve the purpose of exemplifying the concepts and skills within the GE***. ***Because this is not an assessment document, the items should not be copied and used for assessment purposes.*** The examples are provided to support teachers in understanding the essence of the GE, and are a suggested resource that could be used for the teaching and learning of a concept.

Appendix A contains web links when items were drawn from outside sources. Many of these sites contain additional materials which may be useful in the instructional setting. Appendix B contains the Depth of Knowledge descriptors based on Norman Webb's work, and Appendix C contains the answers and Depth of Knowledge classification for each item. In addition, each item is numbered sequentially in the upper left hand corner for ease of reference to the answers in Appendix C.

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Number and Operations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Mathematics Grade Expectations: Grade 4

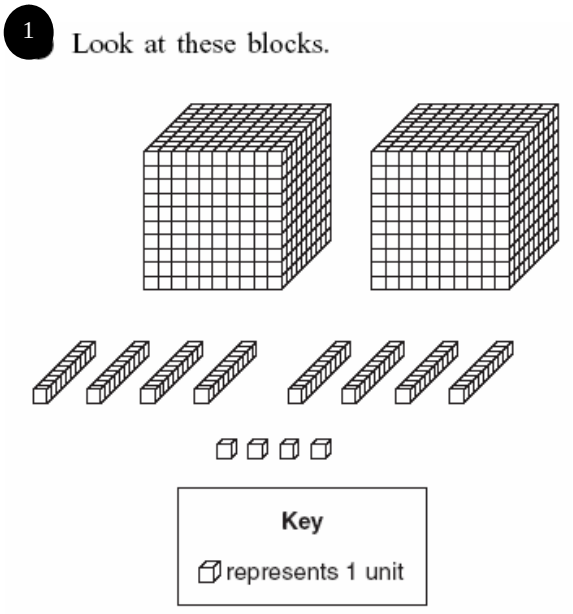
Standard 7.6: Arithmetic, Number, and Operation Concepts

M4: 1 Demonstrates conceptual understanding of rational numbers with respect to:
whole numbers from 0 to 999,999 through equivalency, composition, decomposition, or place value **using models, explanations, or other representations**; and

positive fractional numbers (benchmark fractions: *a*/2, *a*/3, *a*/4, *a*/5, *a*/6, *a*/8, or *a*/10, where *a* is a whole number greater than 0 and less than or equal to the denominator) as a part to whole relationship in area, set, or linear models where the number of parts in the whole are equal to, and a multiple or factor of the denominator; and **decimals** as hundredths within the context of money, or tenths within the context of metric measurements (e.g., 2.3 cm) **using models, explanations, or other representations**.

M(N&O)–4–1

1 Look at these blocks.



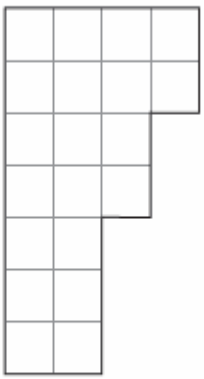
What number do the blocks represent?

A. 284
B. 2084
C. 2804
D. 2840

Demonstrates conceptual understanding of rational numbers with respect to whole numbers through composition or place value using models

NECAP Released Item 2005

2 Using grid paper, Fletcher made this model of the shape of Vermont.



Demonstrates conceptual understanding of rational numbers with respect to positive fractional numbers in area models

He learned that $\frac{3}{4}$ of Vermont is forest. How many ☐ should Fletcher shade gray to represent the fraction of Vermont that is forest?

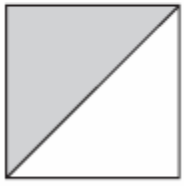
NECAP Released Item 2006

3 **Which square is $\frac{1}{4}$ shaded?**

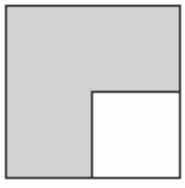
A

B

C



D



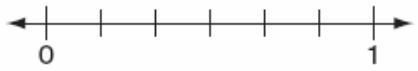
Demonstrates conceptual understanding of rational numbers with respect to positive fractional numbers in area models

MCAS Released Item 2007

M4: 2 Demonstrates understanding of the relative magnitude of numbers from 0 to 999,999 by ordering or comparing whole numbers; and ordering, comparing, or identifying equivalent proper positive fractional numbers; or decimals using models, number lines, or explanations.

M(N&O)–4–2

4 You may use the number line below to answer this question.



Which fractions are in order from least to greatest?

A. $\frac{1}{2}, \frac{2}{3}, \frac{2}{6}$

B. $\frac{1}{2}, \frac{2}{6}, \frac{2}{3}$

C. $\frac{2}{6}, \frac{2}{3}, \frac{1}{2}$

D. $\frac{2}{6}, \frac{1}{2}, \frac{2}{3}$

Demonstrates conceptual understanding of the relative magnitude of numbers by identifying equivalent proper positive fractional numbers using number lines

NECAP 2006 Released Item

M4: 3 Demonstrates conceptual understanding of mathematical operations by describing or illustrating the relationship between repeated subtraction and division (no remainders); the inverse relationship between multiplication and division of whole numbers; or the addition or subtraction of positive fractional numbers with like denominators using models, number lines, or explanations.

M(N&O)–4–3

5 The △ and □ are different numbers that make this sentence true.

△ × 8 = □

Which other number sentence must be true?

A. □ ÷ 8 = △

B. 8 ÷ □ = △

C. △ ÷ □ = 8

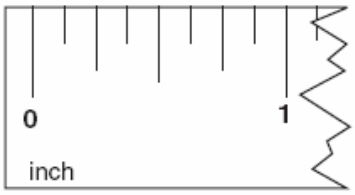
D. 8 ÷ △ = □

Demonstrates conceptual understanding of mathematical operations by illustrating the inverse relationship between multiplication and division

NECAP Practice Test

6

Part of a ruler is shown below.



Demonstrates conceptual understanding of mathematical operations by adding positive fractional numbers with like denominators using models

Rachel is making a bead necklace. The table below shows the width of the two types of beads Rachel is using.

Type of Bead	Width of Each Bead
Round	$\frac{2}{8}$ inch
Square	$\frac{3}{8}$ inch

What is the width of 2 round beads and 1 square bead placed side by side?

NECAP Released Item 2005

M4: 4 Accurately solves problems involving multiple operations on whole numbers or the use of the properties of factors and multiples; and addition or subtraction of decimals and positive proper fractions with like denominators. (Multiplication limited to 2 digits by 2 digits, and division limited to 1 digit divisors.)

(IMPORTANT: *Applies the conventions of order of operations where the left to right computations are modified only by the use of parentheses*.)

M(N&O)–4–4

7

Quinn used a total of 56 apples to make pies. Each pie was made with the same number of apples. How many pies could Quinn have made?

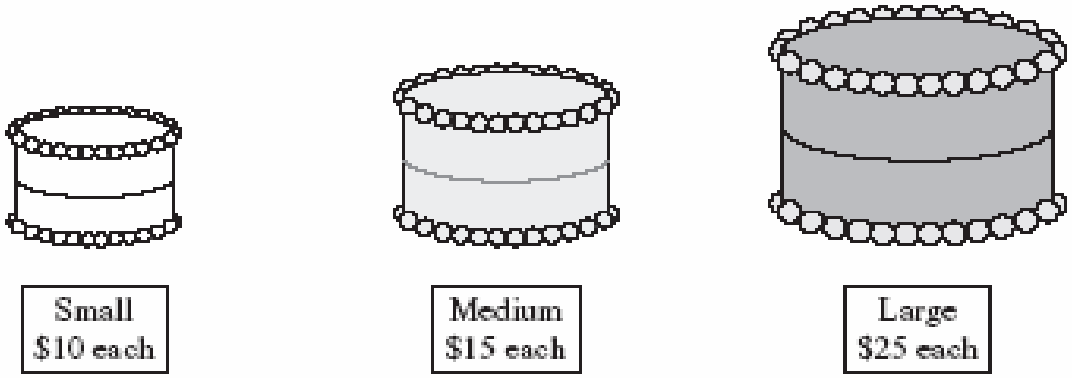
A. 3
B. 7
C. 9
D. 11

Accurately solves problems involving the use of the properties of factors and multiples

NECAP Released Item 2006

8

Hudson's Bakery sells cakes in three different sizes – small, medium, and large. The picture below shows the cost of each size of cake at the bakery.



a. Wilma bought 1 small cake and 2 medium cakes. What was the total cost of the cakes Wilma bought? Show your work or explain how you got your answer.

b. Justin has $85.00 to spend on cakes. What is the greatest number of cakes he can buy with exactly $85.00? Show your work or explain how you got your answer.

c. Sheila bought a group of cakes that cost a total of $70.00. At least 2 of the cakes she bought were different sizes. List a group of cakes that Sheila could have bought. Show your work or explain how you got your answer.

MCAS Released Item 2006

Accurately solves problems involving multiple operations on whole numbers

M4: 5 No **M4: 5** at this grade level

M4: 6 Mentally adds and subtracts whole numbers through twenty and multiplies whole numbers through twelve **with accuracy.**

M4: 7 Estimates and evaluates the reasonableness of solutions appropriate to grade level.

M4: 8 Applies properties of numbers (odd, even, factor, multiple, remainders, composition/decomposition) **to solve problems and to simplify computations.**

9

Which statement is true?

Applies properties of numbers (factors) to solve problems

A The only factors of 8 are 1 and 8.

B The only factors of 9 are 1 and 9.

C The only factors of 10 are 1 and 10.

D The only factors of 11 are 1 and 11.

CA Released Task 2007

Geometry and Measurement

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills. We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes. Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world. In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

Standard 7.7: Geometry and Measurement Concepts

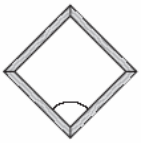
M4: 9 Uses properties or attributes of angles (number of angles) **or sides** (number of sides, length of sides, parallelism, or perpendicularity) **to identify, describe, or distinguish among** triangles, squares, rectangles, rhombi, trapezoids, hexagons, or octagons; or classify angles relative to 90^{o} as more than, less than, or equal to.

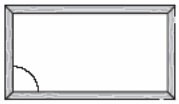
M(G&M)–4–1

10 Recognizes symmetrical figures and uses symmetry to identify and classify figures.

Mr. Hanson makes frames. In which frame is the identified angle more than 90°?

A. 

B. 

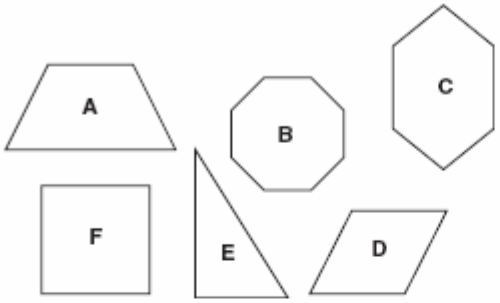
C. 

D. 

Uses properties or attributes of angles or sides to classify angles relative to 90° as more than, less than or equal to.

NECAP Released Item 2005

11 Mr. Grimaldi asked his class to identify a mystery shape from these shapes.



Uses properties or attributes of angles (number of angles) or sides (number of sides) to identify, describe, or distinguish among triangles, squares, rectangles, rhombi, trapezoids, hexagons, or octagons;

He gave the class these two clues.

Clue 1: The mystery shape has fewer than 5 sides.

Clue 2: The mystery shape does **not** have any 90° angles.

a. Using the clues, the class determined that the mystery shape is one of two shapes. What are those two shapes?

b. Pick one shape you identified in part a. What additional clue could Mr. Grimaldi give as Clue 3 that would identify **only** that shape as the mystery shape? Explain your reasoning.

Later, Mr. Grimaldi added this shape to the 6 shapes above.

c. Write one or more clues that could identify this new shape as the only mystery shape.

NECAP Practice Test

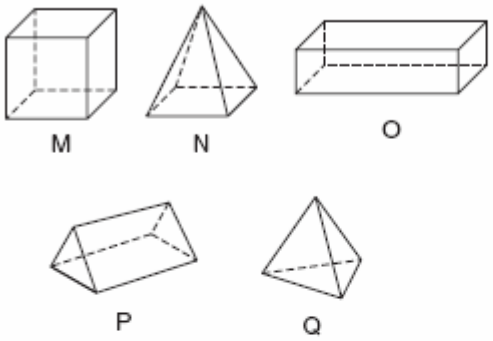
M4: 10 No **M4: 10** at this grade level

M4: 11 Uses properties or attributes (shape of bases or number of lateral faces) **to identify, compare, or describe three-dimensional shapes** (rectangular prisms, triangular prisms, cylinders, or spheres).

M(G&M)–4–3

Identifies components (faces, edges, and vertices) of three dimensional shapes (cubes and rectangular prisms).

12 Jack and Diane each picked a mystery solid from the ones shown below.



a. Here are the clues to Jack's mystery solid.

Clue 1: The mystery solid is a prism.

Clue 2: The mystery solid has 5 faces.

Which solid is Jack's mystery solid?

b. Here are the clues to Diane's mystery solid.

Clue 1: The mystery solid is a prism.

Clue 2: All of its faces are the same shape.

Which solid is Diane's mystery solid?

Uses properties or attributes to identify or describe three-dimensional shapes

NECAP Released item 2006

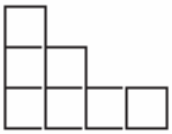
M4: 12 Demonstrates conceptual understanding of congruency by matching congruent figures using reflections, translations, or rotations (flips, slides, or turns), or as the result of composing or decomposing shapes using models or explanations.

M(G&M)–4–4

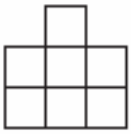
13 Draw a rectangle in your Student Answer Booklet. On your drawing use a dotted line to show where to divide the rectangle into one triangle and one trapezoid.

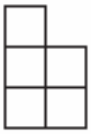
NECAP Practice Test

14 Leah made this figure using grid paper.

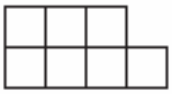


Without any gaps or overlaps, which figure fits together with Leah's figure to form a rectangle congruent to the rectangle below?

A. 

B. 

C. 

D. 

Demonstrates conceptual understanding of congruency as the result of decomposing shapes using models

Demonstrates conceptual understanding of congruency by matching congruent figures using reflections, translations, or rotations (flips, slides, or turns), using models or explanations

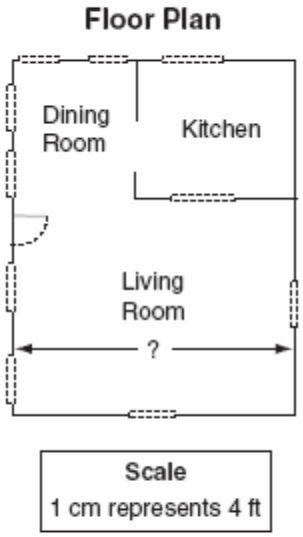
NECAP Released Item 2006

M4: 13 Demonstrates conceptual understanding of similarity by applying scales on maps, or applying characteristics of similar figures (same shape, but not necessarily the same size) to identify similar figures, or to solve problems involving similar figures. Describes relationships using models or[sc] explanations.

M(G&M)–4–5

15

Use your ruler to answer this question. The picture below shows the floor plan of part of a house.



What is the width of the living room?

A. 9 feet
B. 14 feet
C. 20 feet
D. 25 feet

Demonstrates conceptual understanding of similarity by applying scales on maps

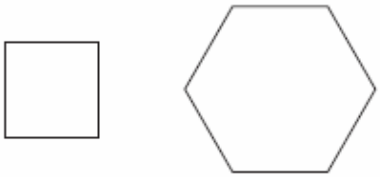
NOTE: The floor plan is not drawn to scale in this format. Refer to the Released Items on the DOE website for the correct scale.

NECAP Released Item 2005

M4: 14 Demonstrates conceptual understanding of perimeter of polygons, and **the area of** rectangles, polygons, or irregular shapes on grids using a variety of models, manipulatives, or formulas. Expresses all measures using appropriate units.

M(G&M)–4–6

16 Karen used toothpicks to make the two shapes shown below.

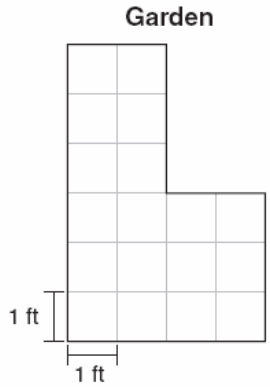


She used a total of 24 toothpicks to make the square. She made the hexagon so that its sides are the same length as the sides of the square. How many toothpicks did Karen use to make the hexagon?

Demonstrates conceptual understanding of perimeter of polygons using models

NECAP Released Items 2006

17 The picture below shows the shape of a garden.



a. What is the perimeter of the garden? Label your answer with the correct unit of measure.

b. What is the area of the garden? Label your answer with the correct unit of measure.

c. What is the perimeter of a rectangle that has the same area as the garden? Show your work or explain how you know. Label your answer with the correct unit of measure.

Demonstrates conceptual understanding of area and perimeter of irregular shapes on grids using models. Expresses all measures using appropriate units

NECAP Released Item 2005

M4: 15 Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands. (Benchmarks in Appendix B.) M(G&M)–4–7

18

Jake is 52 inches tall. Which of the following measurements is the same as 52 inches?

A. 4 feet 2 inches
B. 4 feet 4 inches
C. 5 feet 2 inches
D. 5 feet 4 inches

Makes conversions within systems when solving problems

MCAS Released Item 2007

M4: 16 Determines elapsed and accrued time to the ¼ hour.

19

Three classes will go to the book fair at Carter Elementary School. The first class will arrive at the book fair at the time shown on the clock below.

a. At what time will the first class arrive at the book fair?

Each class will spend 30 minutes at the book fair and then leave. The second class will arrive at the book fair as the first class leaves, and the third class will arrive as the second class leaves.

b. What is the total amount of time that all three classes will spend at the book fair? Show or explain how you got your answer.

c. What time will it be when the third class **leaves** the book fair? Show or explain how you got your answer.

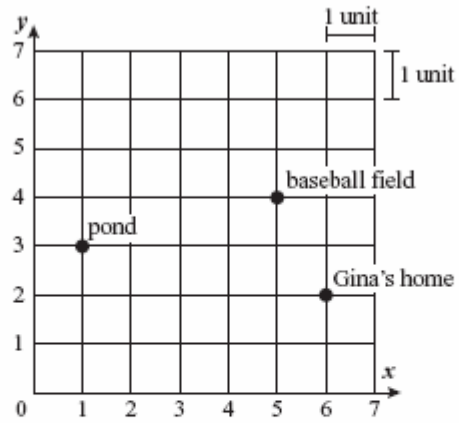
MCAS Released Item 2007

M4: 17 No **M4: 17** at this grade level

M4: 18 Solves problems using the Cartesian coordinate system (Quadrant 1) **to** locate coordinates and to represent data from tables.

20

The points on the grid below represent the locations of Gina's home, a pond, and a baseball field. The grid lines represent the streets in Gina's neighborhood.



a. Write the ordered pair that best represents the location of Gina's home on the grid.

b. Moving along the grid lines, the shortest distance from Gina's home to the baseball field is 3 units. Moving along the grid lines, what is the shortest distance, in units, from Gina's home to the pond? Show or explain how you got your answer.

c. Moving along the grid lines, the shortest distance from Gina's home to her school is 7 units. Write an ordered pair that could be the location of her school. Show or explain how you got your answer.

Solves problems using the Cartesian coordinate system (Quadrant 1) to locate coordinates

MCAS Released item 2006

Functions and Algebra

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students will progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

Standard 7.8: Functions and Algebra Concepts

M4: 19 Identifies and extends to specific cases a variety of patterns (linear and nonlinear) represented in models, tables or sequences; and writes a rule in words or[sc] symbols to find the next case.

M(F&A)–4–1

21 Ramon wrote the pattern shown below.

6, 18, 30, 42, 54, . . .

Which of the following is a rule for Ramon's pattern?

A. add 6
B. add 12
C. multiply by 3
D. multiply by 6

What is the next number in Ramon's pattern?

Identifies and extends to specific cases a variety of patterns (linear) and writes a rule in words to find the next case.

MCAS Released Item 2007

M4: 20 Demonstrates a conceptual understanding of linear relationships ($y = kx$) **as a constant rate of change** by identifying, describing, or comparing situations that represent constant rates of change.

22 The table below shows the amounts of money needed to park for different numbers of minutes. Each 6 minutes costs the same amount of money.

Parking Amounts

Amount of Money	Time (in minutes)
5 cents	6
10 cents	12
15 cents	18
20 cents	24
25 cents	30

How much money will it cost to park for 60 minutes?

Demonstrates conceptual understanding of linear relationships as a constant rate of change by identifying situations that represent constant rates of change

MCAS Released Item 2007

M4: 21 Demonstrates conceptual understanding of algebraic expressions by using letters or symbols to represent unknown quantities to write simple linear algebraic expressions involving any one of the four operations; or by evaluating simple linear algebraic expressions using whole numbers.

M(F&A)–4–3

23

Mr. Farrell uses the expression $50 \times n$ to calculate the weight (in pounds) of n crates. How much would 20 crates weigh?

A. 70 pounds
B. 100 pounds
C. 700 pounds
D. 1000 pounds

Demonstrates conceptual understanding of algebraic expressions by evaluating simple linear algebraic expressions using whole numbers

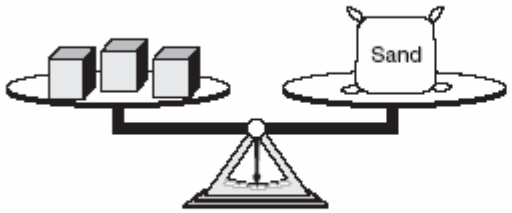
NECAP Released Item 2005

M4: 22 Demonstrates conceptual understanding of equality by showing equivalence between two expressions using models or different representations of the expressions, by simplifying numerical expressions where left to right computations may be modified only by the use of parentheses [e.g., $14 – (2 \times 5)$] (expressions consistent with the parameters of M(F&A)–4–3), and by solving one-step linear equations of the form $ax = c$, $x \pm b = c$, where a, b, and c are whole numbers with $a \neq 0$

M(F&A)–4–4

24

The scale shown below is balanced.

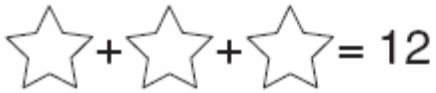


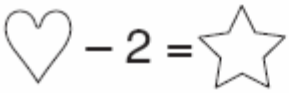
Demonstrates conceptual understanding of equality by showing equivalence between two expressions or different representations of the expressions

The bag of sand weighs 18 pounds. Each of the cubes has the same weight. How many pounds does one cube weigh?

NECAP Released Item 2005

25 In these number sentences, each star represents the same number and the heart represents a different number.





Demonstrates conceptual understanding of equality by solving one-step linear equations of the form $x \pm b = c$, where a, b, and c are whole numbers with $a \neq 0$

a. What number does the star represent? Show your work or explain how you know.

b. What is the value of one star plus two hearts? Show your work or explain how you know.

NECAP Released Item 2006

Data, Statistics and Probability

Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

Standard 7.9: Data, Statistics, and Probability Concepts

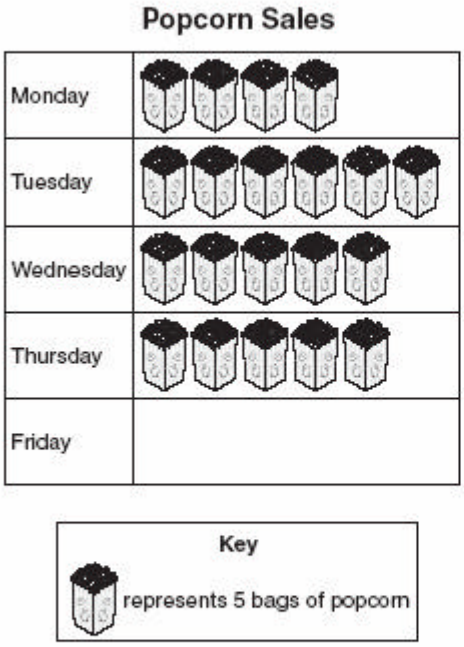
M4: 23 Interprets a given representation (line plots, tables, bar graphs, pictographs, or circle graphs) to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.

(IMPORTANT: *Analyzes data consistent with concepts and skills in M4: 24.*)

M(DSP)–4–1

And (tally charts, frequency charts, line graphs, Venn diagrams).

26 Students sold a total of 130 bags of popcorn during a popcorn sale. Marge is making this pictograph to show the number of bags of popcorn sold on each day of the sale. The sales for Friday are missing.



How many  does Marge need to add to the pictograph to show Friday's sales? Show your work or explain how you know.

Interprets a given representation (pictograph) to answer questions related to the data to solve problems

NECAP Released Item 2006

M4: 24 Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using measures of central tendency (median or mode), or range.

M(DSP)–4–2

27

Mona has 5 dogs.

- The oldest dog is 10 years old.
- The median age of the dogs is 6 years.
- The mode of the dogs' ages is 2 years.

Give one possible list of the ages of Mona's dogs from the youngest to the oldest.

Analyzes distributions in data by determining or using measures of central tendency (median or mode)

NECAP Released Item 2005

M4: 25 Organizes and displays data using line plots, bar graphs, tally charts and frequency charts, or tables to answer question related to the data, to analyze the data to formulate or justify conclusions, or to make predictions.

(IMPORTANT: *Analyzes data consistent with concepts and skills in M4: 24.*)

28

Sara asked 35 students in her school to tell her their favorite color. The tally chart below shows the students' favorite color.

Create a bar graph using the information in this tally chart to show students' favorite color.

Write 1 question which could be answered using the data in your bar graph. Answer the question you wrote.

Favorite Colors

Blue	𝍸 |||
Green	𝍸 ||
Red	𝍸 |||
Yellow	𝍸
Purple	𝍸 |

Organizes and displays data using bar graphs to answer questions related to the data.

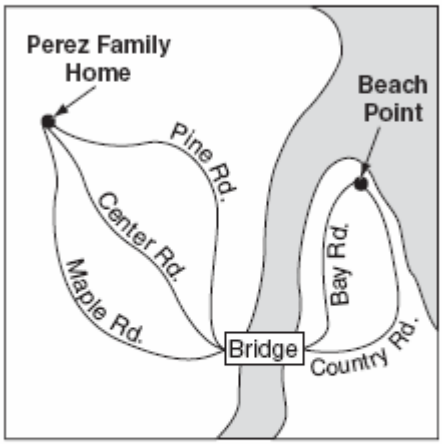
Vermont Department of Education

M4: 26 Uses counting techniques to solve problems in context involving combinations or simple permutations (e.g., given a map, determines the number of paths from point A to point B) using a variety of strategies (e.g., organized lists, tables, tree diagrams, or[sc] others).

M(DSP)–4–4

29

The diagram below shows all the roads the Perez family can take from their home to Beach Point.



How many different ways can the Perez family drive from their home to Beach Point?

A. 2
B. 3
C. 5
D. 6

Uses counting techniques to solve problems in context involving combinations or simple permutations (e.g., given a map, determines the number of paths from point A to point B)

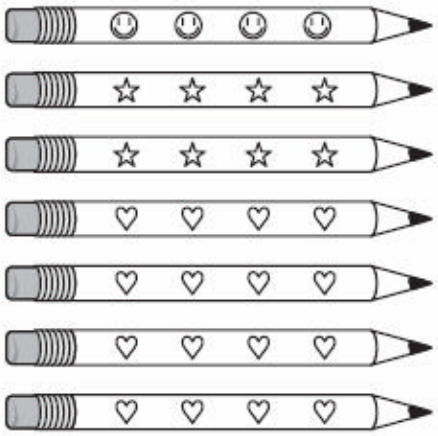
NECAP Released Item 2006

M4: 27 For a probability event in which the sample space may or may not contain equally likely outcomes, determines the theoretical probability of an event and expresses the result as part to whole (e.g., two out of five).

M(DSP)–4–5

30

Ms. Wang has the pencils shown below in her desk.



She took one pencil out of her desk without looking.

What is the probability that Ms. Wang took out a pencil with stars on it?

For a probability event in which the sample space may or may not contain equally likely outcomes, determines the theoretical probability of an event and expresses the result as part to whole (e.g., two out of five).

MCAS Released Item 2007(Modified)

M4: 28 In response to a teacher - or student-generated question or hypothesis, collects appropriate data, organizes the data, displays/represents the data, analyzes the data to draw conclusions about the questions or hypothesis being tested.

(IMPORTANT: *Analyzes data consistent with concepts and skills in M4: 24.*)

31

The students in your class want to organize an ice cream party for all students in fourth grade. Your teacher believes most of the students would like vanilla ice cream. Collect, organize, display and analyze data from all the students in grade four to determine if your teacher is correct. Based on your data analysis, determine the 3 most popular ice cream flavors.

In response to a teacher - or student-generated question or hypothesis, collects appropriate data, organizes the data, displays/represents the data, analyzes the data to draw conclusions about the questions.

Vermont Department of Education

M4: 29 Uses experimental probability, records the outcomes, and describes the likelihood of an event as a value from 0 through 1 (for events that are certain to occur) written as either a ratio or as part to whole (e.g., 7 out of 10).

32

Find the experimental probability of winning games of chess after playing 10 games. Write your prediction and proceed with playing 10 games of chess. Record the number of times you win and the number of times you lose. Determine the experimental probability of winning games of chess after playing 10 games. Offer a different prediction and play another round of 10 games.

Vermont Department of Education

Standard 2.5: Mathematical Dimensions, Standard 7.10: Mathematical Problem Solving and Reasoning—Applications

M4: 30 Demonstrate understanding of mathematical problem solving[2] and communication through:[3]

- **Approach & Reasoning**—The reasoning, strategies, and skills used to solve the problem;
- **Connections**—Demonstration of observations, applications, extensions, and generalizations;
- **Solution**—All of the work that was done to solve the problem, including the answer;
- **Mathematical Language**—The use of mathematical language in communicating the solution;
- **Mathematical Representation**—The use of mathematical representation to communicate the solution; and
- **Documentation**—Presentation of the solution.

[2] Problem-solving situations are mathematical problems that reflect the levels of mathematics in the Grade Level Expectations.
[3] See *Vermont Elementary and Middle Level Mathematics Portfolio Scoring Guide* for additional information.

APPENDIX A

National Library of Virtual Manipulatives. **http://nlvm.usu.edu/en/nav/vlibrary.html**

California Department of Education. **http://www.cde.ca.gov/ta/tg/sr/resources.asp**

Massachusetts Department of Education. **http://www.doe.mass.edu/mcas/testitems.html**

Pennsylvania Department of Education.
http://www.pde.state.pa.us/a_and_t/cwp/view.asp?a=108&q=103267

New England Common Assessment.
http://reporting.measuredprogress.org/NECAPpublicNH/relitem.aspx

New York State Assessment. **http://www.nysedregents.org/testing/eleminttests.html**

NCTM. **http://www.nctm.org/resources/elementary.aspx**

TIMSS. **http://www.nctm.org/resources/elementary.aspx**

Delaware Student Testing Program. **http://www.doe.state.de.us/AAB/DSTP_items.html**

Vermont Department of Education

APPENDIX B:
Implied Cognitive Demand and Depth of Knowledge

A fundamental criterion used to develop the NECAP GLEs and GSEs is that the expectations should explicitly indicate cognitive demand (how content interacts with process) and that there should be a mix of cognitive demand levels at all grades. One should not assume that students at lower grades do less cognitively demanding work. The cognitive demand or depth of knowledge required by an expectation or an assessment item is related to the number and strength of connections of concepts and procedures that a student needs to make to produce a response, including the level of reasoning required along with self-monitoring. Furthermore, there are additional factors that influence cognitive demand including contextual requirements, language, the number and variety of representations, requirements for generalizations to new situations, and the opportunity to learn.

It is important to note that depth of knowledge is not synonymous with difficulty. As an example, solving a multi-step linear equation with variables on both sides may be a difficult task for middle school students; however, the task can be solved by applying a standard procedure making the task of low complexity.

The NECAP states believe that expectations and assessments should be aligned in terms of their cognitive complexity. That is, the cognitive complexities of the assessment items should match those of the standards (what students are expected to know and be able to do). To ensure this alignment, the NECAP states have adopted Norman L. Webb's (senior researcher with the Wisconsin Center for Educational Research) Depth of Knowledge classification system. Norman Webb's system is based on four levels of classification. The full descriptions of each level are given on pages 4 and 5. The levels can be summarized as follows.

Level 1 Recall
Level 2 Skill/Concept
Level 3 Strategic Thinking
Level 4 Extended Thinking

The NECAP states, together with a committee of educators, analyzed the GLEs and GSEs for their implied cognitive demand. All aspects of each expectation were analyzed and the implied cognitive demand levels were recorded. One of the charges of the NECAP test item review committees is to ensure that assessment items align not only with the expectations but also with their implied cognitive demands. The range of cognitive demands for each GLE and GSE is summarized in Table 1. It should be noted that the highest level listed for each GLE and GSE should be thought of as a "ceiling" not a "target". A NECAP goal is to write items that cover the range of the levels indicated and not just the highest level. If one assesses only at the "target" level, all GLEs with a level 3 (for example) as their "ceiling" would only be assessed at level 3. This would potentially have two negative impacts on the assessment: 1) The assessment as a whole would be too difficult, and 2) important information about student learning along the achievement continuum would be lost. To the extent possible, each GLE and GSE should be assessed at the "ceiling" and at least one level below the "ceiling" in order to provide additional diagnostic information to educators. Furthermore, Table 2 shows an example of an expectation and how the different aspects of the expectation interact with Table 1.

Vermont Grade Expectations
Implied Cognitive Demand and Depth of Knowledge

	K	1st	2nd	3rd	4th	5th	6th	7th	8th	HS
Arithmetic, Numbers and Operations										
M1	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2
M2	1,2	1,2	1	2	2	2	2	2	2	
M3	2	2	1,2	2	2	2,3	2,3	2,3		
M4	1	1,2		1,2,3	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3
M5	1	1	1,2							
M6		1	1	1	1	1	1			
M7	2	2	2	2	2	2	2	2	2	2
M8		1,2	2	2,3	2,3	2,3	2,3	2,3	2,3	2,3
Geometry and Measurement Concepts										
M9	1	1	1,2,3	1, 2	1,2	1,2	1,2	1,2	1,2,3	1,2,3
M10								1,2	1,2,3	
M11		1	1,2	2	1,2	1,2	1,2	2,3		2,3
M12				2	2	1,2	1,2	1,2		
M13					2	2	1,2	1,2,3	1,2,3	1,2,3
M14			1,2	1,2	1,2	1,2	1,2,3	1,2,3	1,2,3	1,2,3
M15	1	1	1	1	1,2	1,2	1,2	1,2	1,2	1,2
M16	1	1	1	1	1	1				
M17								1,2	1,2	1,2
M18	1	1	2	2	2	2	2			
Functions and Algebra Concepts										
M19	1,2	1,2	2	2	2	2	2,3	2,3	2,3	2,3
M20	2	1,2	1,2	2	2	2	1,2	1,2,3	1,2,3	1,2,3
M21					1	1	1,2	1,2	1,2	1,2
M22	1,2	1,2	1	1,2	1,2	1,2	1,2	1,2	1,2,3	1,2,3

Data, Statistics, and Probability Concepts										
M23	2,3	2,3	1,2,3	1,2,3	1,2, 3	1,2,3	1,2,3	1,2,3	2,3	2,3
M24	2,3	2,3	2,3	2,3	2,3	2,3	2,3	2,3	2,3	1,2,3
M25	2,3	2,3	2,3	1,2	2,3	1,2	2,3	2,3	2,3	2,3
M26			2	2	2,3	2,3	2,3	2,3	2,3	1,2,3
M27		1,2	1,2	1,2	1,2	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3
M28	3,4	3,4	3,4	3,4	3,4	3,4	3,4	3,4	3,4	3,4
M29				2,3	1,2,3	1,2,3	2,3	2,3	2,3	2,3
M30	3,4	3,4	3,4	3,4	3,4	3,4	3,4	3,4	3,4	3,4
Note: GEs assessed on the NECAP are shaded gray.										

Depth of Knowledge Descriptors for Mathematics
Norman L. Webb
March 28, 2002

Mathematics Depth of Knowledge Levels

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain," "describe," or "interpret" could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Mathematics Depth of Knowledge Levels continued

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

References

New Hampshire, Rhode Island, and Vermont Department of Education. (2004). *Draft Tri-State New England (TSNE) Mathematics Test Specifications*. New Hampshire, Rhode Island, and Vermont Department of Education.

Webb, L. Norman. (2002). Depth of Knowledge Levels for Four Content Areas.

Webb, L. Norman. (1997). Criteria for Alignment of Expectations and Assessments in Mathematics and Science Education. Research Monograph No. 8. Council of Chief State School Officers

APPENDIX C:
Answers and DoK Codes for Examples

Item Number	Answer	Depth of Knowledge
1	B	2
2	15	2
3	A	1
4	D	2
5	A	2
6	7/8	2
7	B	1
8	$40 – Answers will vary 7 Small and 1 Medium 2 large, 2 Small	2
9	D	1
10	D	1
11	A+D - Answers will vary – exactly 2 90 degree angles	3
12	P – M or O	2
13		2
14	B	2
15	C	1
16	36	2
17	20– 18 - Answers will vary	2
18	B	2
19	9:15 – 90 minutes – 10:45	2
20	6,2 – 6 – Answers will vary	3
21	B	2
22	$.50	2
23	D	1
24	6 pounds	2
25	4 - 16	2
26	6 – Answers will vary	3
27	2,2,6,7,10 Answers will vary	3
28	Answers will vary	3
29	D	2
30	2 out of 7	2
31	Answers will vary	3
32	Answers will vary	2